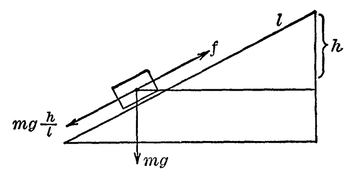
SOLUTION BY B. L. LIBBY, Ann Arbor, Mich.

Using the notations indicated in the figure, we have the resultant force acting upwards

$$f - mg \cdot \frac{h}{l}.$$



The acceleration required is therefore given by

$$a = \frac{fl - mgh}{ml}$$

Hence $dl/dt = (fl - mgh/ml)t$, and $l = (fl - mgh/2ml)t^2$, from which

$$t^2 = 2ml^2/(fl - mgh).$$

Differentiating, we have

$$t\frac{dt}{dl} = \frac{mfl^2 - 2m^2ghl}{(fl - mgh)^2}.$$

The usual test for a minimum for t gives

$$l = \frac{2mgh}{f} \quad \text{and} \quad t = 2m\sqrt{\frac{2gh}{f}}.$$

Also solved by HORACE OLSON and P. PEÑALVER.

347. Proposed by R. D. CARMICHAEL, Indiana Univ.

Show that the differential equation

$$9\left(\frac{d^2y}{dx^2}\right)^2\frac{d^5y}{dx^5} - 45\frac{d^2y}{dx^2}\frac{d^3y}{dx^3}\frac{d^4y}{dx^4} + 40\left(\frac{d^3y}{dx^3}\right)^3 = 0$$

remains unchanged when the variables x and y undergo any projective transformation.

SOLUTION BY A. M. HARDING, University of Ark.

Any projective transformation can be reduced to a succession of integral transformations of the form

$$x = aX + bY + c, \qquad y = a_1X + b_1Y + c_1, \tag{1}$$

combined with the particular transformation

$$x = \frac{1}{X}, \qquad y = \frac{Y}{X}.$$

Hence it will be sufficient to show that the given equation remains invariant under each of these transformations. From (1) we find

$$y' = \frac{a_1 + b_1Y'}{a + bY'}, \; y'' = \frac{(ab_1 - a_1b)Y''}{(a + bY')^3}, \; y''' = \frac{(ab_1 - a_1b)[(a + bY')Y''' - 3b(Y'')^2]}{(a + bY')^5},$$

$$y^{\text{iv}} = \frac{(ab_1 - a_1b)[(a + bY')^2Y^{\text{iv}} - 10b(a + bY')Y''Y''' + 15b^2(Y'')^3]}{(a + bY')^7},$$

$$y^{\text{v}} = \frac{(ab_1 - a_1b)[(a + bY')^3Y^{\text{v}} - 15b(a + bY')^2Y''Y^{\text{iv}} - 10b(a+bY')^2(Y''')^2 + 105(a + bY')b^2(Y'')^2Y''' - 105b^3(Y'')^3]}{(a + bY')^9}$$

From (2) we find

$$y' = Y - XY', \quad y'' = X^3Y'', \quad y''' = -X^4(3Y'' + XY'''),$$

$$y^{\text{iv}} = X^5[12Y'' + 8XY''' + X^2Y^{\text{iv}}]$$

$$y^{\text{v}} = -X^6[60Y'' + 60XY''' + 15X^2Y^{\text{iv}} + X^3Y^{\text{v}}].$$

It can be easily shown by substitution that each transformation leaves the given equation invariant.

Also solved by J. W. CLAWSON and GEO. W. HARTWELL.

349. Proposed by C. N. SCHMALL, New York City.

If $y = a \cos(\log x) + b \sin(\log x)$, eliminate the constants a and b and obtain the equation

$$\frac{x^2d^2y}{dx^2} + \frac{xdy}{dx} + y = 0.$$

SOLUTION BY C. C. STECK, New Hampshire State College.

By differentiating the equation $y = a \cos(\log x) + b \sin(\log x)$, we obtain

$$\frac{xdy}{dx} = -a \sin(\log x) + b \cos(\log x).$$

Differentiating this and multiplying the resulting equation by x, we get

$$\frac{x^2d^2y}{dx^2} + \frac{xdy}{dx} = -a \cos(\log x) - b \sin(\log x) = -y.$$

From the last two equations readily follows the desired result

$$\frac{x^2d^2y}{dx^2} + \frac{xdy}{dx} + y = 0.$$

Also solved by M. E. GRABER, J. B. SMITH, J. W. CLAWSON, P. PEÑALVER, C. HORNUNG, ELMER SCHUYLER, A. M. HARDING, W. W. BEMAN, A. L. MCCARTY, H. L. SLOBIN, CLIFFORD N. MILLS, FRANCIS RUST, I. A. BARNETT, F. C. REISLER, G. W. HARTWELL, S. W. REAVES, RICHARD MORRIS, ALBERT R. NAUER, BARNEM LIBBY, and WALTER C. EELLS.

351. Proposed by C. N. SCHMALL, New York City.

In the ellipse $(x^2/a^2) + (y^2/b^2) = 1$ are given e the eccentricity, and the angle φ which the normal at any point P (on the curve) makes with the major axis. If R is the radius of curvature at P prove that

$$R = \frac{a(1 - e^2)}{(1 - e^2 \sin^2 \varphi)^{\frac{3}{2}}}.$$